CHAPTER 3

Modeling Interactions between the Environment and the Economy

This chapter presents a regionalized computable general equilibrium (CGE) model in which Brazil is subdivided into regions compatible with the major administrative subdivisions adopted by the Brazilian government: Amazon, Northeast, Center-West, and South/Southeast. For the Legal Amazon, the following processes are considered: (1) conversion of forests to cleared land (which depends on agents' economic decisions), and (2) transformation of land from cleared land to grassland, and (3) subsequent transformation from grassland to an unproductive state.

The starting point for the regionalized CGE model is a nationwide model developed in 1995–96 as an ongoing collaborative effort between the International Food Policy Research Institute (IFPRI) and the Brazilian National Development Bank (BNDES).

The regional model has two components: a CGE model, which represents the behavior of economic agents, and a land transformation model, which is a simplified representation of biophysical processes affecting land productivity. This chapter begins with a brief survey of the approaches that have been adopted to address deforestation issues using CGE models, followed by a description of the characteristics of the CGE model used for this research and a description of how the biophysical processes are represented.

General Equilibrium Models: From Theory to Practice

In general equilibrium theory, the goal is to formulate a model of simultaneous equilibrium in competitive markets for all commodities that is a precise logical representation of the interaction of consumers and producers. The simplest form of general equilibrium model is the input-output model pioneered by Leontief (1941). In the static input-output model, there is no joint production, only one technique exists for producing each output, and all technologies have constant returns to scale. Input requirements for each unit of output are given by fixed coefficients, and final demand is exogenous. The appeal of this approach is its conceptual simplicity and the tractability afforded by computing equilibrium prices by matrix inversion. The scheme of using matrices to keep track of flows between sectors persists to this day within more complex models of general equilibrium. Isard and Kaniss (1973) give a good account of the uses and shortcomings of the input-output model.

Activity analysis generalizes the production structure by representing it in terms of alternative activities, that is, combinations of inputs and outputs where the ratios between inputs and outputs are fixed in each instance but vary between activities. Joint production is permitted in activity analysis, and there may be more than one activity producing the same output

(Koopmans 1951; Dorfman, Samuelson, and Solow 1958). Within the linear programming environment, prices are assumed exogenous, multiple consumers are not permitted, and the model contains no price distortions. Under these conditions it could be proved that shadow prices coincided with market prices

CGE modeling originated with the work of Johansen (1960). He was the first to introduce a feedback from production levels and endogenous prices to final demand. Johansen solved the general equilibrium model for growth rates by linearizing the model in logarithms and applying matrix inversion techniques. He introduced nonlinear neoclassical substitution possibilities in production and consumption and endogenous determination of market-clearing product and factor prices. The Johansen approach was further developed by Dixon et al. (1982) in their multisectoral ORANI model for the Australian economy. Darwin et al. (1995) and Hertel (1990, 1997) are also in the same tradition.

A technique that is becoming widely adopted is to recast equilibrium problems as *mixed complemetarity problems* (MCP). The MCP is a fundamental problem in optimization that encompasses many of the continuous optimization problems, such as quadratic programming and nonlinear programming, as special cases. It is useful for expressing systems of nonlinear inequalities and equations. A common representation of an MCP has two components: the first represents a set of underlying conditions defined by a system of nonlinear equations, and the second constitutes the complementarity conditions that are only applied to some of the variables and functions. The problem can be specified as follows: given a nonlinear function

$$F : \mathbf{R}^n \to \mathbf{R}^n, \text{ find an } x \in \mathbf{R}^n$$

let I and J be a partition over $\{1, 2, \ldots, n\}$ such that

$$F_I(x) = 0, \quad x_I \text{ free, and}$$

$$F_J(x) \geq 0 \quad \perp \quad x_J \geq 0$$

Where the perpendicular notation "⊥" signifies that, in addition to the stated inequalities, the equation $x^T{}_J F_J\ (x) = 0$ also holds. For existence and uniqueness of the solution to this problem, see Ferris and Kanzow (1998).

The connection between traditional optimization techniques in economics and this wider problem class was first made by Cottle and Dantzig (1970). A natural connection was also the use of mathematical programming methods in partial equilibrium models pioneered by Samuelson (1949). For a review of papers on the formulation and solution of computable equilibrium problems such as MCP, see Manne 1985; Cottle, Pang, and Stone 1992; Ferris and Pang 1997.

An area that has received wide attention in the field of complementarity problems has been the development of robust and efficient algorithms for solving large-scale applications efficiently. Along with the research in the design of algorithms came the linkage of these algorithms with programming model languages such as the General Algebraic Modeling System (GAMS). The research results to be presented here have been obtained using the PATH solver (available with GAMS), which uses a search method that is a generalization of a line search technique (Dirkse and Ferris 1995).

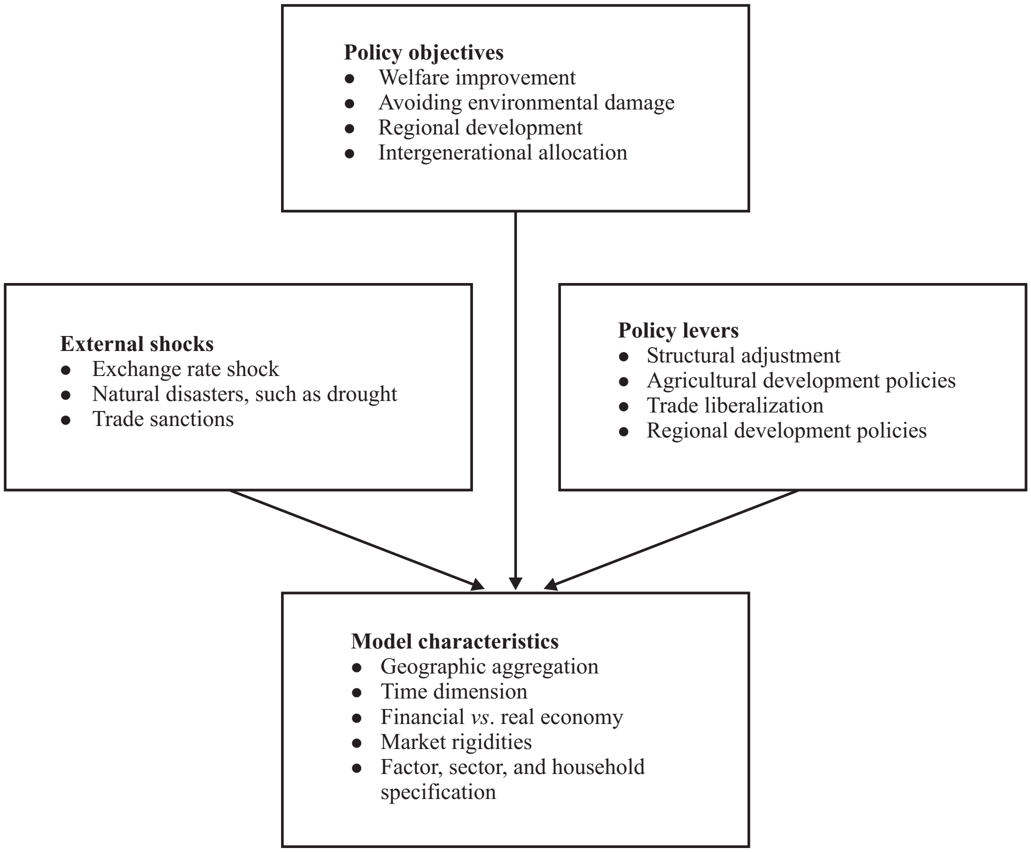
Modeling Approaches

CGE models have been categorized from analytical through stylized to applied (Robinson 1989). Analytical and stylized numerical models explore the magnitude of the effects of particular causal mechanisms and usually do not provide sufficient detail to analyze and support specific policy recommendations. Applied models consist of a more detailed specification of the institutional side of the country-specific economy under study. Although applied models allow for detailed analysis, there is a danger of concealing the basic causal mechanisms of the model without enhancing its empirical significance, a fact that should be kept in mind when choosing detailed features for an applied model specification (Devarajan, Lewis, and Robinson 1994).

In the domain of the applied models, the detailed nature of CGE models is driven by concerns about policy objectives, external shocks being imposed, and the policy tools being considered to meet the objectives and face the exogenous shocks (Figure 3.1). The combination of these three factors determines the adequate geographic and sectoral aggregations and indicates the appropriate way of representing time. More importantly, the underlying theoretical paradigm will also be affected by these factors.

Although the core of CGE models is neoclassical microeconomic theory, combined with the multisectoral intermediate input links adapted from input-output models, modelers have had to abandon some of the strict neoclassical assumptions in order to meet the imperfections of the actual economies under observation. Instead of perfect competition with perfectly flexible prices and free product and factor mobility, applied CGE models often incorporate

Figure 3.1 Factors affecting the appropriate structure of a CGE model



structural rigidities, which seek to capture nonneoclassical behavior, macro imbalances, and institutional rigidities typical of developing economies.[9] The relevant theoretical features that describe macro adjustment, political economy, uncertainty, incomplete markets, and temporary equilibrium are not directly incorporated into the models, but imposed through ad hoc constraints, which are not directly related to the agents' endogenous rational behavior.

Geographic Aggregation

CGE models can be divided into subnational, single-country, and multicountry models. All are open-economy models and incorporate the "rest of the world" as an integral component that permits the consideration of worldwide capital and commodity flows and consequently their influence on the economy under observation. The analytical focus of the study to be carried out determines the geographic aggregation to be applied. Single-country models are used for analyses with a single, national focus. Multicountry models are used to address questions such as global trade liberalization, regional trade agreements, interregional migration, and climate change issues.[10]

Although less common, the focus is sometimes at a subnational level. In such cases one can choose among a spectrum of options for capturing the regionality inside the country. If there are several economically distinguishable regions to be fully represented, a separate CGE model can be constructed for each region connected by flows of factors and commodities as in the multicountry models (Robinson, Hoffman, and Subramanian 1994) .[11] Lofgren and Robinson (1999) present a spatially disaggregated national CGE model that incorporates interregional and national-regional feedbacks to analyze the spatial impacts of economic policies. On the other hand, if regionality is relevant only to a subset of the economic process, such as the presence of a regionally specified activity or factor or both it may be sufficient to maintain a national specification for the model as a whole, while distinguishing the few relevant regional characteristics (Coxhead and Warr 1991; Coxhead and Jayasuriya 1994).

Another reason to model at the subnational level is that the interest is in a natural resource base that is geographically defined. In such cases modeling the single regions, for example, a watershed, may be the appropriate solution (Mukherjee 1996). Isard et al. (1998) present a detailed overview on applied general interregional equilibrium models.

Specification of Time

If the focus of the analysis is comparative statics, the appropriate approach is a single-period model in which all time flow is collapsed into the time before and after an exogenous, unexpected shock. In this case time plays a limited role, agents' expectations are assumed static, only impacts on flows are considered and not impacts on stocks, and the timeframe for adjustment is generally captured by the mobility of factor markets expressed in the model closure.[12] While this approach may appear to be an

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[9] These deviations from the Walrasian paradigm and their corresponding methodological problems are criticized in Srinivasan 1982; Bell and Srinivasan 1984; and Shoven and Whalley 1984.

[10] For surveys on this matter refer to Shoven and Whalley 1992; Brown 1992; Goldin, Knudsen, and van der Mensbrugghe 1993; OECD 1990.

[11] In this case care has to be taken because the different regions share a common exchange rate.

[12] Usually in the short-term factors reflect limited intersectoral mobility in the labor markets and none in the capital markets (1 year); in the medium-term labor has full mobility but capital is still fixed (2–4 years); finally, in the long-term both factors are mobile (5–10 years).

oversimplification, it is a useful indicator of the order of magnitude of the impact of a shock or policy measure over an approximate timeframe. At the opposite extreme of the single-period model, there are perfect foresight, intertemporally specified CGE models. This type of model is appropriate when the main focus is the transition path associated with a shock. This interest may arise from a concern with the distribution of income over generations, associated for example with an aging population, or from inefficiencies that could arise from fluctuations in the tax burdens over time. In cases like these, dynamic CGE and other models are best suited to compare the long-term gains of a policy and its short-term costs.

Between the two extremes of static and rational expectations models there is a broad spectrum of options. A deeper treatment of time in CGE models reflects mainly on the stock-flow relationships and the assumptions about agent behavior over time. First, if a model is to be intertemporal, an equation of motion has to be specified to update the factor stocks for labor through population growth, for capital through investment, and for the natural resource base through degradation/regeneration. Second, one must represent the agents' expectations concerning prices and projected incomes. The latter point can be dealt with in a variety of ways ranging from backward-looking expectations (which can be solved recursively) to perfect foresight models (Dixon and Parmenter 1996). The recursive approach is often considered the appropriate choice for capturing the transition path and, in fact, it is often used for forecasting purposes. There are two approaches to macro forecasting in a CGE framework: the first option is to rely on CGE-generated macro implications, and the second is to rely on exogenously supplied macro forecasts, using the CGE model to carry out structural forecasts (Dixon and Parmenter 1996).[13]

The perfect foresight approach is appealing for its model-consistent expectations. Forward-looking models will generally have four distinguishing characteristics. First, consumption is represented as part of life-cycle behavior of consumers. Household behavior is determined by the maximization of an additively separable, time-invariant, intertemporal utility function subject to a lifetime intertemporal budget constraint. Households can be represented as being constituted by *overlapping generations or as infinitely lived agents.*[14] Second, firms are assumed, first, to maximize their market value, which is equal to the present value of their dividend streams, and second, to face imperfect capital mobility due to adjustment costs (*q*-theory).[15] Third, the government faces an intertemporal budget constraint, and if the government is allowed to run deficits, the debt path is endogenously determined (Pereira 1988; Pereira and Shoven 1988). Finally, the balance of trade and international capital flows have to be specified; not much has been done in this area, and most models assume balanced trade and no capital flows.[16]

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[13] Forecasting with CGE-generated macro scenarios has not been very successful. When using an external macro forecast, compatibility with the CGE model is ensured by endogenizing variables like total factor productivity and the propensity to save (see Dixon and Parmenter 1996 for more on this matter).

[14] Early work on the overlapping generations dynamic models was done by Auerbach and Kotlikoff (1983); Ballard (1983); Ballard and Goulder (1985); for the infinitely lived agent approach see Bovenberg 1985 and Andersson 1987.

[15] Examples of such firm behavior specifications can be found in Bovenberg 1984; Summers 1985; Goulder and Summers 1987; and Devarajan and Go 1998.

[16] Exceptions are Andersson 1987 and Erlich, Ginsburgh, and Van der Heyden 1987.

Infinite-horizon formulations face severe computational problems when used in applied models. Another drawback of this type of approach is that the baseline to which the simulations will be compared is a balanced growth path (which may or may not occur in reality). Finally, the discount factor, which is generally specified exogenously, will often generate an unrealistic sequence of savings rates (Ginsburg 1994). A good compromise is to build a two-period intertemporal model for a policy measure or shock that takes place during the first period (Erlich, Ginsburgh, and Van der Heyden 1987; Persson 1995).

For an early survey on dynamic CGE models (concentrating on tax policy evaluation), see Pereira and Shoven (1988). In the final part of a book edited by Mercenier and Srinivasan (1994), four contributions by different authors are concerned with modeling intertemporal trade-offs. Azis (1997) compares the impacts of economic reform on rural-urban welfare in a static and a dynamic framework and thereby focuses not only on the economic objectives of the study, but also on the differences of its results with respect to the different methodological approaches. In this vein, Abbink, Braber, and Cohen (1995) demonstrate under what assumptions a simple static CGE model can be extended to a dynamic CGE specification, and they apply both versions simultaneously. Very few applications show explicit interest in and specification of intertemporal aspects of the development process, such as the multisectoral CGE with overlapping generations and intertemporal optimization presented by Keuschnigg and Kohler (1995).[17] Another example is Go (1995), who highlights the intertemporal trade-offs of tariff reforms when examining the sensitivity of investment and growth to external shocks and adjustment policy. Dynamic CGE models are very useful in order to simulate the overall economic development path of an economy. Diao, Yeldan, and Roe (1998) construct a dynamic applied general equilibrium model of a small open economy in order to investigate the transition path and convergence speed of out-of-steady state growth paths in response to trade policy shocks.

Environmental Externalities and Natural Resource Use

Since the 1970s there have been numerous applications of CGE modeling to energy and natural resource issues. Models relating to energy range from those with highly disaggregated specifications of the energy sector, allowing for substitution between energy sources and specifying different demand types, to those focusing more on the rest of the economy, containing a simplified representation of the energy sector.[18] The latter generally focus on the differential impact of a natural-resource boom or crisis on the tradable and nontradable sides of the economy (Benjamin1996; Martin and van Wijnbergen 1986). As an example of the former, Hudson and Jorgenson (1974) constructed an econometric general equilibrium model that captured the interrelationships between energy policies and

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[17] Keuschnigg and Kohler (1995) analyze the dynamic effects of trade liberalization in Austria.

[18] Surveys for the disaggregated approach are Devarajan 1988; Bergman 1988; and Bhattacharyya 1996.

economic growth. The authors examined the role of energy taxes in promoting conservation and how to employ the price system to adapt to changes in the availability of energy resources.

The role of taxation to compensate for environmental externalities and its general equilibrium effects are fertile topics for CGE analysis both because the societal costs of such a tax can be estimated through its effect on prices and income (positive analysis), and because optimal taxes may be computed (normative analysis). Jorgenson and Wilcoxen (1990) examine the costs to the economy of emissions regulation and the implications of a carbon tax.[19] For a period there was debate over the so-called “double-dividend” hypothesis, postulating that if the revenue from emission charges is used to reduce the tax on wage income then positive employment effects can result in “second-best” situations with preexisting distortions (Terkla 1984). While this debate has not been resolved, the hypothesis seems to hold only in the short run and under restrictive assumptions (Carraro, Galeotti, and Gallo 1996; Scholz 1998). An interesting development, as the theory of market incentives evolved, was to include markets for tradable emission permits where the equilibrium prices of permits reflect the marginal costs of emission control (Bergman 1991). In reality, the problem with this approach is that a tradable permit program, compared with taxation, has no revenue-raising mechanism to cover the high monitoring costs.[20]

Because of the local and global externalities associated with tropical deforestation, the results presented in the previous paragraphs are important in the context of the research described in this report; however, deforestation occurs mostly on privately owned land. This implies that the economic agent owning the land will view it as an input to production, either agricultural or for timber where externalities are not taken into consideration, or maybe for conservation if externalities are fully internalized. It is therefore important to understand how land as a factor of production is represented in CGE models.

Land is a heterogeneous factor in agricultural production and this poses interesting challenges and possibilities from a modeling standpoint. The productive possibilities of a given hectare of land depend on soil type, drainage, declivity, and climate. These characteristics affect the yield for any specific crop given labor and capital inputs, and therefore determine (along with considerations of the other factors) the most suitable economic activity on a parcel of land. A CGE model focusing on agriculture must manage to capture the constraints on supply response arising from land heterogeneity. Perhaps the simplest method available is to segment the land market along land types that can be put to similar uses. For example, rice and corn can be substituted in production if the land is good, but a producer cannot switch from mediocre pasture to producing rice or corn on that land. This approach implies that activities are either perfectly substitutable or not substitutable at all. A more flexible approach is that adopted by Robidoux et al. (1989) who also differentiate between land types and land uses, but the land types substitute imperfectly in the production of a given crop.[21] In both approaches the land-specific rental rate must be equal across uses. An alternative

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[19] See Bhattacharyya (1996) for a survey on the use of CGE for environmental policy analysis.

[20] Revenues can be generated by auctioning off permits, but this one-time inflow will not cover monitoring costs.

[21] The authors of this study on Canada specify constant elasticity of substitution (CES) aggregator functions that combine land types, each of which is used to some degree in each crop.

approach is that adopted by Hertel and Tsigas (1988); they specify a transformation function that takes aggregate farmland as an input and employs it in various uses based on the elasticity of transformation and relative rental rates.

Unlike labor and capital, land is geographically immobile. Regional or climatic differences can be expressed in a number of ways. If farmland is represented as an aggregate input as in Hertel and Tsigas (1988), regionality is difficult to incorporate unless it is embedded in the crop specification. To portray regionality appropriately, land types have to be differentiated along geographic or climatic lines as in Darwin et al. (1995). Land classes are then employed differentially across sectors according to current patterns of production.

This section concludes with an overview of the use of CGE models to analyze issues relating to forestry and deforestation. Following Xie, Vincent, and Panayoutou (1996), CGE models dealing with forest resources can be broadly classified into three groups. The first group consists of applications of standard CGE models that include a forestry sector alongside the other production sectors of the economy (Cruz and Repetto 1992; Coxhead and Jayasuriya 1994; Coxhead and Shively 1995). The second group considers the dynamic nature of forests' reaction to economic processes and resolves the intertemporal forest harvesting problem by modeling a steady state (Dee 1991; Thiele and Wiebelt 1992; Wiebelt 1994; Thiele 1994). The steady-state specification assumes that foresters choose an economically optimal harvest pattern. The limitation of this approach for deforestation in tropical areas such as Brazil is, first, that logging is closer to an extractive process, as opposed to a sustainable, managed forest operation. Second, deforestation is driven mostly by land clearing for agricultural purposes. The third group of models differentiates land uses and types and introduces property rights considerations (Persson and Munasinghe 1995; Persson 1995). They include logging and squatter sectors and therefore markets for logs and cleared land. The model adopted in this paper extends the approach of Persson and Munasinghe (1995) to include land degradation as a feedback mechanism into the deforestation process. A more in-depth review of CGE model applications to deforestation can be found in Kaimowitz and Angelsen (1998).

In their comprehensive review of economic models of deforestation spanning theoretical constructs and scales, Kaimowitz and Angelsen (1998) note some commonality in findings-that ease of access to forest and to long-distance trade paths as well higher agricultural and timber prices or lower rural wages increase deforestation rates. However, problems at each scale of analysis contribute to what Kaimowitz and Angelsen highlight in their review as inconclusive or ambiguous findings about the effects on deforestation of macroeconomic forces, population and migration, changes in productivity and input markets (including land markets and tenure security), and household wealth—or poverty. Since that review, Barbier (2001) has collected papers analyzing deforestation that emphasize economic modeling techniques or that incorporate spatial features and institutional factors (including placement of parks and reserves).

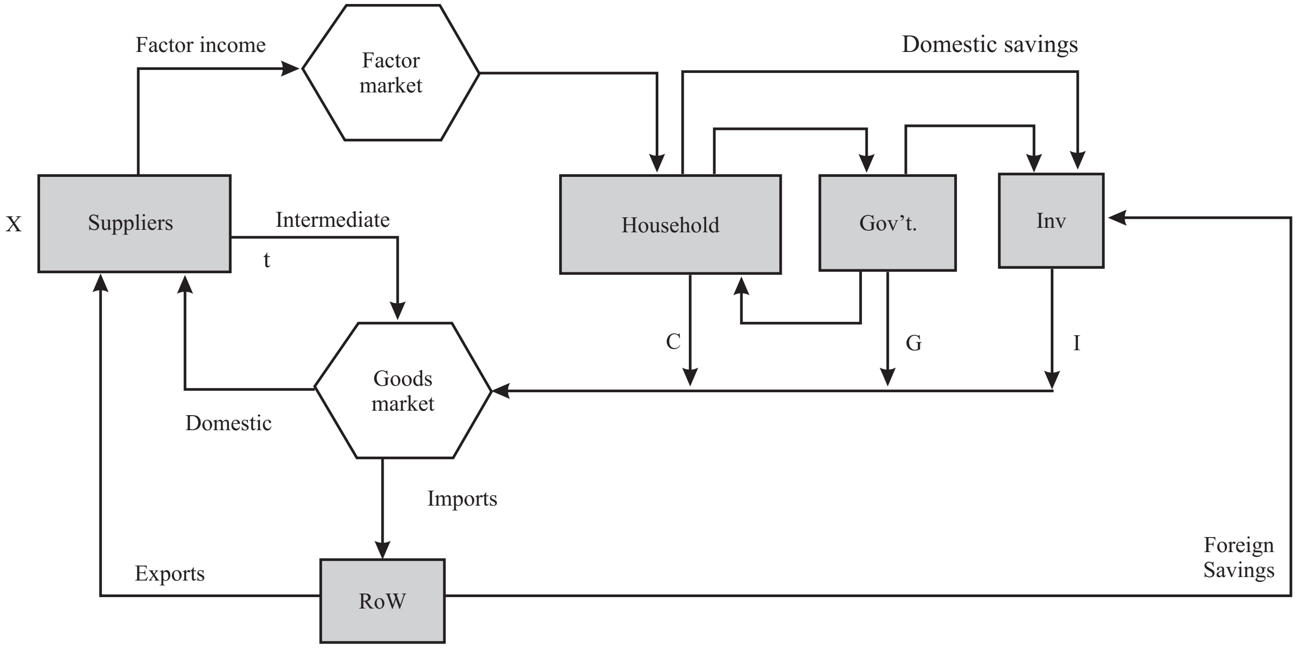
CGE Model Structure: A Primer

In the standard approach to CGE models, one first distinguishes between different agents, such as producers, consumers, and government, and then between goods and factors and the associated markets through which agents interact. The behavioral assumptions of agents are rooted in conventional microeconomic theory: producers maximize profits subject to certain technological constraints (nonincreasing-returns-to-scale production functions) while consumers maximize utility subject to budget constraints, all within the framework of competitive markets. Equilibrium in this type of model is characterized by a set of prices and levels of production such that the market demand equals supply for all commodities. Factors are either fully utilized with flexible market-clearing wages or rent, or alternatively, the wage of a factor has a lower bound below which there is excess supply of that factor. The intersectoral allocation of factors is endogenously determined. The model is specified as a system of nonlinear simultaneous equations. The basic elements of the model can be represented by the circular flow diagram of the economy presented in Figure 3.2. The starting point for the development of this model is a standard CGE model as described in Dervis, de Melo, and Robinson (1982), and the structure of the model draws most directly on Robinson, Kilkenny, and Hanson (1990) and Robinson (1990).

Factor incomes generated by production activities are divided among households in factor-specific shares representing factor ownership. Total household income is used to pay taxes, save, and consume. Government revenue comes from the collection of ad valorem direct taxes and indirect taxes. Government transfers income to households, and expenditure is a fixed share of total absorption. The rest of the world supplies imports and demands export goods. Brazil is treated like a "small country" in the sense that the export demands and import supplies that it faces are infinitely elastic at prevailing prices (with the exception of coffee and sugar).

The macro system constraints (or macro closures) determine the manner in which the accounts for the government, the rest of the world, and savings and investment are brought into balance. On the spending side of the savings-investment balance, nominal

Figure 3.2 CGE structure showing the circular flow of income



Notes: RoW is rest of world, C is consumption, G is government, and I is investment.

aggregate investment is either a fixed share of total absorption, or it adjusts according to the households' savings rate. On the savings side, if investment is fixed, the average household saving rate adjusts to achieve the level of savings that matches the exogenously specified level of investment. In the government account, total nominal government expenditure is a fixed share of total absorption, and government saving is endogenously determined by the model. Foreign savings is exogenous and the exchange rate adjusts the current account balance.

Model Characteristics

In the modeling approach adopted here, a regionalized CGE model is developed in which Brazil is subdivided into regions compatible with the major administrative subdivisions adopted by the Brazilian government: Amazon, Northeast, Center-West, and South/Southeast.[22] For the Amazon the following processes are considered: (1) conversion of forest to cleared land (depends on agents' economic decisions), (2) transformation of cleared land to grassland, and (3) subsequent transformation from grassland to unproductive states.[23]

The overall model has two components: the CGE model, representing the behavior of economic agents, and the land transformation model, which is a simplified representation of biophysical processes affecting land productivity.

The model allows for two-way trade (cross-hauling) assuming that imports and domestic demand as well as exports and domestic supply are imperfect substitutes (Armington assumption). Producers maximize profits with respect to their nested constant elasticity of substitution (CES) production functions, and households maximize utility with respect to Cobb-Douglas household consumption.[24]

The model is nonfinancial because it does not explicitly include money and asset markets. This choice is based on the assumption that the types of shock considered (changes in the nominal exchange rate, transportation costs, and agricultural technologies) affect most directly the real side of the economy, such as quantities of production and commodities consumed, rather than monetary effects, inflation, and interest rates. While the above hypothesis is somewhat unrealistic in certain situations, the lack of data on the functioning of financial markets necessary to integrate supply and demand variables for money and assets is a limiting factor in modeling financial intermediation of the savings and investment process.[25]

The model is static and solves for a new equilibrium within a single period, given a specified external shock, internal shock, or policy change. The previous section on dynamic CGE models provides some insight into the pros and cons of examining change over time via CGE models used for different analytical purposes. The underlying motivation for choosing a comparative statics approach is that the issues of interest here do not depend on intertemporal optimization by agents, whether it be firms' investment behavior or households' life-cycle saving patterns. The scenarios to be analyzed involve one-time shocks or policy

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[22] For the definition of the Amazon region adopted in this report, refer to footnote #4.

[23] The methods adopted could be used to study subsequent regeneration processes through secondary forest growth or planting improved pasture.

[24] The reason for specifying consumption as being Cobb-Douglas is that the income shifts for most of the simulations are sufficiently small that a unitary income elasticity for the components of final demand will not affect the outcome of the simulations for the variables in this report.

[25] See Bourguignon, Branson, and de Melo (1992) for an example of the integration of asset portfolio behavior of macroeconomic models in Tobin's tradition into a CGE model.

measures to which the structure of the economy must adjust in order to return to equilibrium. In terms of expectation models, the shock is a "surprise," requiring adjustments to reestablish the macro balance of the economy.

A complete CGE model also includes a number of closure rules. Closure rules place aggregate constraints on the economic activity simulated in the CGE model. They pertain to how the major macroeconomic accounts (government, trade, labor and capital accounts) adjust to regain equilibrium in response to changes in economic activity. When specifying the model, the system will be overdetermined and one of the constraints of the model must be relaxed to find a solution. Choosing a particular closure rule means precisely deciding which constraint should be dropped. There is no clear-cut theoretical justification for the choice of a particular closure rule except the modeler's general view of an underlying macroeconomic behavior that is assumed exogenous to the CGE model. The closure rules have been shown to have a considerable impact on model structure and the policy conclusions reached (Lysy 1982; Dewatripont and Michel 1987; Robinson 1991). The macroeconomic closure rules of the model and the specification of its factor markets (presented in detail in a later section) will determine the short-, medium-, or long-term character of the model.

The present approach incorporates a number of distinctive model features in order to capture the mechanisms underlying deforestation and agricultural development in a complex setting like Brazil. First, the research is centered on the role of land as a factor of production; therefore, different land classes, with distinct productive possibilities, are specified based on geographic location and vegetative cover. Land in each region is differentiated according to its land type on the basis of cover: (1) forested land, (2) arable land, (3) grassland/pasture, and (4) degraded land.[26] Second, an important characteristic of the marketing process in developing countries with insufficient infrastructure in transport and communication services is the prevalence of high transport and marketing costs. The present approach takes into account this particular characteristic of the economy by incorporating specific marketing margins that are associated with each of the four regions present in the model. This specification allows for detailed analysis of both the economy-wide and regional effects of investment to improve infrastructure. Third, the model incorporates a detailed regional specification of agricultural technologies in the form of multi-output-production functions. The model can therefore take into consideration the ease or difficulty farmers have in shifting production from one crop to another.

The approach is especially useful when considering the impact of technological improvements in agriculture: if an improved technology is not a "substitute" relative to the crops already in production, the impact of technological change will be limited. Fourth, deforestation has been introduced as an explicit economic activity producing cleared land that is demanded by the investment account. For the purpose of this study, this characteristic of the model is of crucial importance because it links agricultural production to the equilibrium demand for deforested land. Demand for deforested land is assumed to be perfectly elastic with the price paid to deforesters determined by the asset value differential between newly

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[26] Weed infestation associated with nutrient depletion exhibits a marked threshold effect in soils of the humid tropics, effectively leading to a succession to grassland, whereby farmers' production possibilities are affected from one year to the next. The fact that the effect on farmers of soil degradation is nonmarginal, even though the underlying process is continuous, justifies the assumption that land conditions for agricultural purposes can be expressed by discrete states.

cleared land and forested land, which, in turn, depends on the difference in land rent and on the biophysical degradation affecting the returns to land over time. Fifth, having differentiated land as a factor of production into forested land, arable land, and grassland, each with distinctive productive possibilities, the model keeps track of the stocks of these different land types by factoring in biophysical degradation that transforms arable land into grassland, and grassland into degraded land.

Land Classification

This research is centered on the role of land as a factor of production. Close attention is paid to feedback effects of different environmental states on the economy. Therefore, the principal criteria for identifying land heterogeneity should be the extent to which economic agents' decisions are affected by different environmental states. At this point, the modeler/researcher is faced with an important decision: can the environmental state be described in discrete terms or should it be represented as a continuous process? In other words, does an economic agent react to step-wise or continuous variations in resource quality? This is an important decision from a methodological standpoint because it entails different modeling approaches.

If agents respond to step-wise changes, then it is appropriate to differentiate the resource into a finite number of states, with each of these states having a well-defined role in the economy's production possibilities. This could apply, for example, to qualitative changes in land conditions: a farmer has different options depending on whether the land is forested, cleared, or infested by weeds. In this case three land states can be defined: forest, cleared, and grassland. These would appear as factors of production in different economic activities (for example, forest land in agro-forestry, cleared land in grain cultivation and pasture, and grassland in pasture). Marginal action by the economic agent cannot alter the state of the land. Alternatively, if the agent's productive possibilities are affected in a continuous fashion by changes in resource quality, then it is necessary to incorporate a continuous variable in the production function for each activity, which affects productive possibilities. Where land has no distinct state, but rather its productivity varies along a spectrum based on nutrient levels, then nutrients would be included in the production function. In this case, a marginal action by the economic agent, such as applying fertilizer, would have an impact on production.

As nutrients are depleted in soils of the humid tropics, weeds move in. Weed infestation associated with nutrient depletion marks the threshold of a succession to grassland. Therefore, farmers' production possibilities are affected from one year to the next. The fact that soil degradation is a nonmarginal effect, even though the underlying process is continuous, justifies the assumption that land conditions for agricultural purposes can be expressed by discrete states.

To better describe the approach taken here, it is useful to define some terms and concepts. The differentiation of land into four *land types* on the basis of cover is shown in Figure 3.3. These distinctions are based on the qualitative characteristics that economic agents perceive as making these factors fit for use in distinct economic activities. For example, if land is covered in forest, farmers are able to extract timber or other forest products, but they cannot use the land to plant annuals or perennials, or for pasture, until the land is cleared. Similarly, if the land is cleared and weed infestation has not begun, the land is classified as arable, and can be used for annuals, perennials, or pasture. If the weed infestation has passed a threshold beyond which annuals and perennials are no longer viable, it is classified as grassland and can only be used for pasture. Degraded land is unproductive land and can only be left fallow.

Figure 3.3 Land transformation/conversion flows

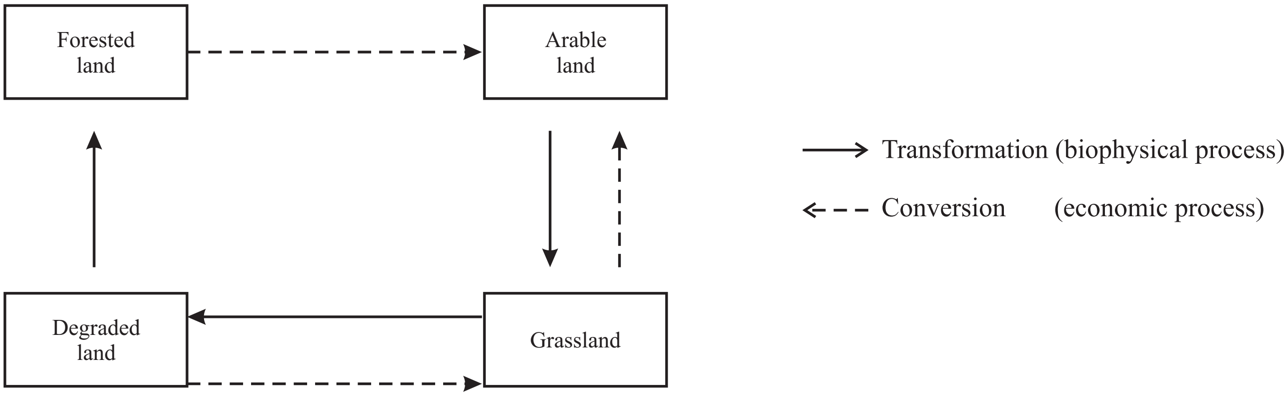


Table 3.1 Mapping of economic activities, commodities produced, and factors used (as adopted in the model)

Activity	Commodities produced	Factors used
Agricultural		
Annuals production	Corn, rice, beans, manioc, sugar, soy, horticultural goods, and other annuals	Arable land, unskilled rural labor, skilled rural labor, and agricultural capital
Perennials production	Coffee, cacao, other perennials	Arable land, unskilled rural labor, skilled rural labor, and agricultural capital
Animal products	Milk, livestock, poultry	Grassland, unskilled rural labor, skilled rural labor, and agricultural capital
Forest products	Nontimber tree products, timber, and deforested land for agricultural purposes	Forest land, unskilled rural labor, skilled rural labor, and agricultural capital
Other agriculture	Other agriculture	Arable land, unskilled rural labor, skilled rural labor, and agricultural capital
Industrial		
Food processing	Food processing	
Mining and oil	Mining and oil	
Industry	Industry	Urban skilled labor, urban unskilled labor, and urban capital - (applies to all sectors)
Construction	Construction	
Trade and transportation	Trade and transportation	
Services	Services	

Land transformations are transitions between land types as a result of physical processes, given certain economic uses. For example, cleared land where rice is cultivated is transformed into grassland.

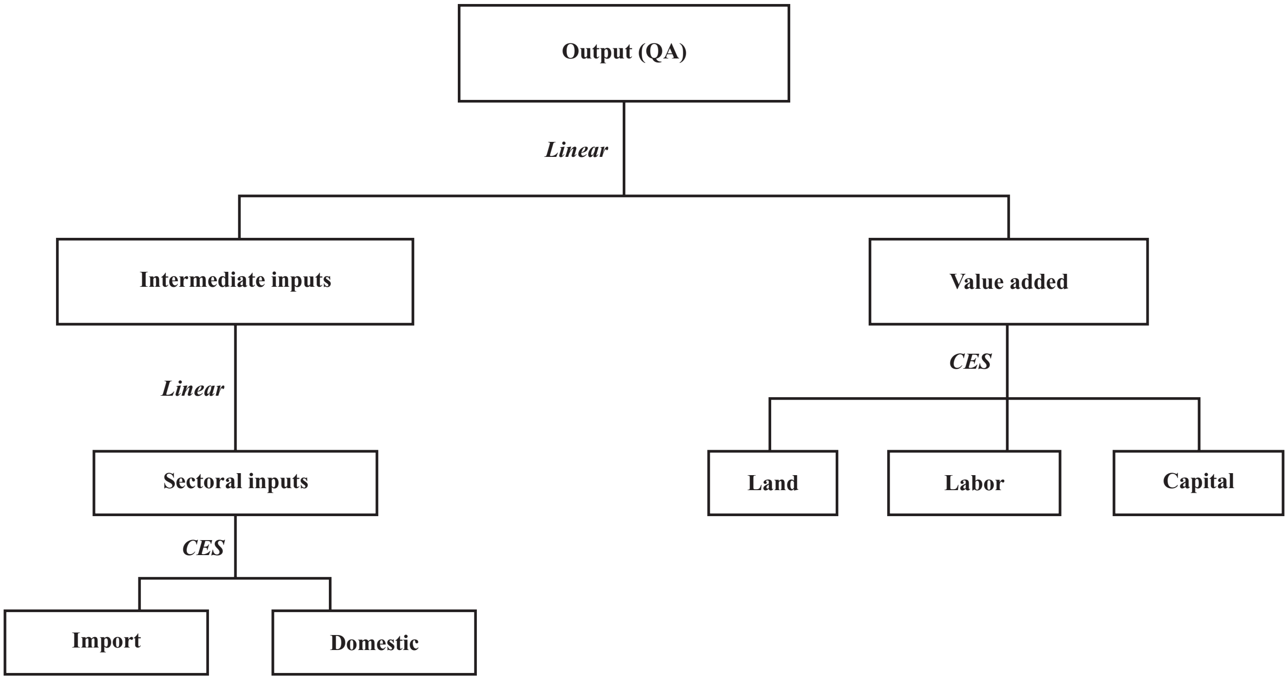
Land conversion describes a transition between two land types brought about intentionally by economic agents. Usually the agent incurs a conversion cost. In the simulations in this study, farmers cut down trees to plant annuals or perennials or for use as pasture.

Representation of Production and Flow of Goods

The activities considered in the model are presented in Table 3.1, along with the factors employed in production and the commodities being produced by these activities.

Agricultural production is disaggregated by region (Amazon, Center-West, Northeast, Rest of Brazil); by activities (annuals, perennials, animal products, forest

Figure 3.4 Sectoral production technology



Notes: CES is constant elasticity of substitution.

products, and other agriculture); and by size of operations (smallholder, large farm enterprise). All factors employed by agriculture are region-specific. Producers are assumed to maximize profits given their technology. Agricultural technologies by sector are specified as two-level production functions assuming separability between the two levels. At the lower level, real value added is a CES function of the primary factors of production; output by activity is a fixed coefficient function of real value added and intermediate inputs. The lower level of production technologies is summarized in Figure 3.4.

The Armington assumption is used to capture the choice between imports and domestic output under imperfect substitutability. All domestic demands (including intermediate demands as shown in Figure 3.4) are for the same composite commodity, with the mix between imports and domestic output determined by the assumption that domestic demanders minimize cost subject to imperfect substitutability, captured by a CES aggregation function. This assumption grants the domestic price system a certain degree of independence from import prices and dampens import responses to changes in the producer environment.

The output of the agricultural activity is transformed, at the second level, into commodities, according to a smooth concave transformation frontier described by a translog function obtained as a production-side analogy of the Almost Ideal Demand System (Deaton and Muellbauer 1980). Convexity of the production set was checked according to Hasenkamp (1976). In effect, each agricultural activity produces a number of agricultural commodities ($QA_a \rightarrow QXAC_{a,c}$) (Figure 3.5). For example, a farm producing annuals in the Amazon may have beans, manioc, and rice as

Table 3.2 Production technology: Substitutability between agricultural commodities

Technology	Commodity 1	Commodity 2	Substitutability
Annuals production	Corn	Rice, beans	Low
	Corn	Manioc	Low-medium
	Corn	Sugar, soy, horticulture, other annuals	Medium-high
	Rice	Beans	Low
	Rice	Manioc	Low-medium
	Rice	Sugar, soy, horticulture, other annuals	Medium-high
	Beans	Manioc	Low-medium
	Beans	Sugar, soy, horticulture, other annuals	Medium-high
	Manioc	Sugar, soy, horticulture, other annuals	Medium
	Sugar	Soy, horticulture, other annuals	High
	Horticultural goods	Other annuals	Medium-high
Perennials production	Coffee	Cacao	High
	Coffee	Other perennials	Medium
	Cacao	Other perennials	Medium-high
Animal products	Livestock	Milk	Medium
	Poultry	Livestock, milk	Medium-high
Forest products	Deforested land (agriculture)	Timber	Low-medium
	Deforested land (agriculture)	Nontimber tree products	High
	Nontimber tree products	Timber	High

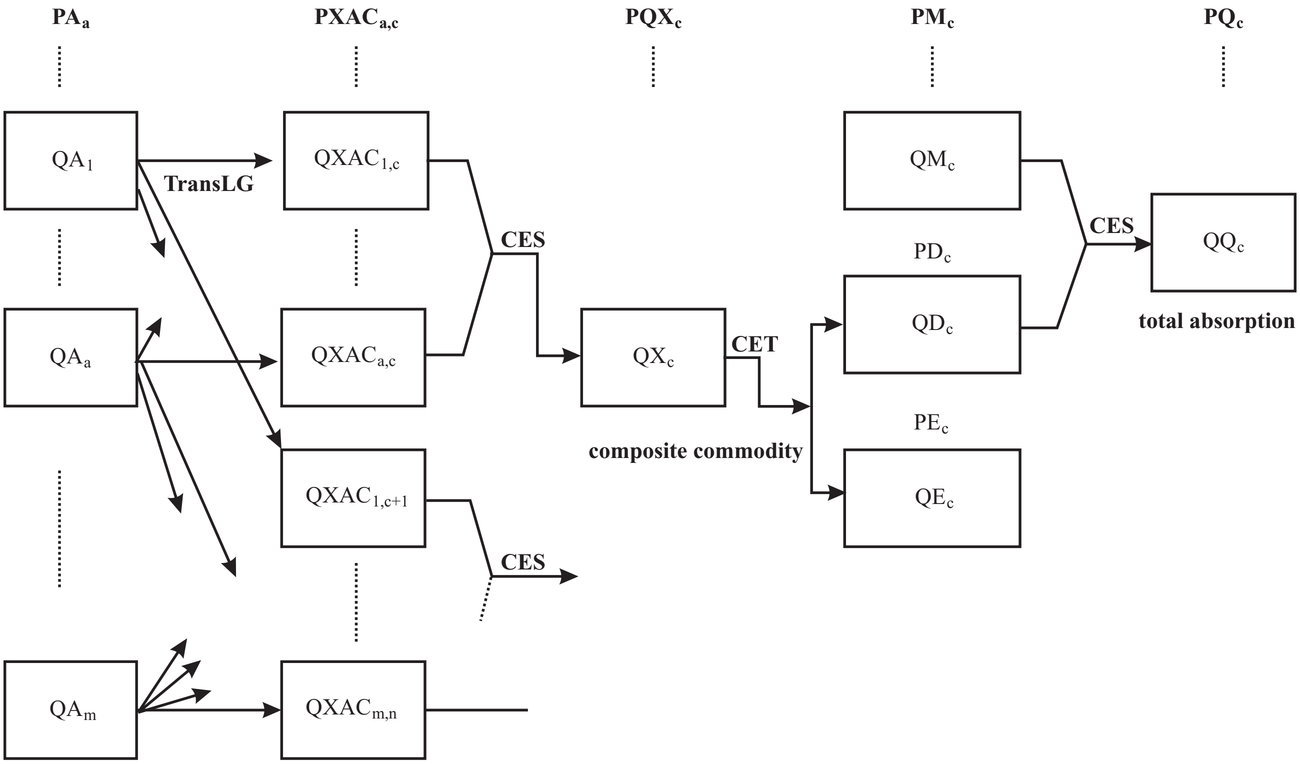
Notes : The elasticity ranges used are: low = 0.1 to 0.3, low-medium = 0.7 to 0.9, medium = 1.0 to 2.0, medium-high = 2.0 to 4.0, and high = 4.0 to 8.0.

output. This specification allows for the possibility that farmers consider certain agricultural commodities as substitutes and others as complements in the production process. The technology captures both price responsiveness, through own-price elasticities, and technological constraints in transforming agricultural output from one commodity to another through substitution elasticities. Values for these elasticities were obtained by distributing a survey among IFPRI and Embrapa researchers with expert knowledge about the production process in Brazilian agriculture. The results are presented in Table 3.2.

The default option assumes high substitutability in production; at the extreme, it approximates the linear programming farm model approach to production by shifting production to the most profitable crop. If, alternatively, the experts believe that farmers weigh price signals with other factors when making this decision, then substitution elasticities would be lower. Possible factors being considered are (1) relative risk associated with the crops, (2) subsistence requirements, (3) crops requiring similar soil characteristics (substitutable) or different soil characteristics (less substitutable), (4) common practice (habit), and (5) whether intercropping is common for two crops (in this case, at the extreme, there would be very low substitutability).

The general flow from production activities to final commodities is presented in Figure 3.5. The notation for price and quantity variables can be found in the next section on model specification. The diagram starts out at the far left following the contribution of different activities (QA_1, ...,QA_m) to the production of a single commodity (QX_c) and, moving to the far right, shows

Figure 3.5 Flow of goods from regional producers to the national composite commodity



Notes: Prices at top are associated with quantities in boxes as goods are transformed to final product to be consumed or exported. CET is constant elasticity of transformation; CES is constant elasticity of substitution; and TransLG is translog multiple output (agriculture only).

how the domestically produced commodity is affected by the export and import markets.

Outputs are treated similarly to the combination of imports and domestic products. Outputs produced by different regional activities, for a same commodity, are treated as imperfect substitutes in demand in a manner that parallels the treatment of imports and outputs of domestic origin ($QXAC_{a,c} \rightarrow QX_c$, using CES aggregation). The result is that regional activities are allowed a degree of independence from their competitors in other regions of Brazil. This protection arises from the fact that they may produce slightly differentiated goods. Even though only one aggregate national market is considered for each commodity (due to data limitations on interregional flows of commodities), it can also be interpreted that the producers are in reality selling to different market segments (for example, along geographic lines). This allows regions facing higher production or transportation costs in the market for a specific commodity to continue producing.

The allocation of domestic output between exports and domestic sales is determined on the assumption that domestic producers maximize profits subject to imperfect transformability between these two alternatives, expressed by a constant elasticity of transformation (CET) function. This assumption grants the national price system

Table 3.3 Definition of variables, parameters, and indices in the ZimCGE model

Equation	Definition
Sets	
A	Activities
C	Commodities
F	Factors
H	Households
LAND	LAND ($\subset$F)
FCON	Factors involved in conversion ($\subset$ LAND)
FMIG	Interregionally mobile factors ($\subset$F)
Parameters	
α	Share of deforestation occuring on tenured land
$cles_c$	Share of consumption allocated by commodity
$dwt_{f_1 f_2}$	Wage differential threshold for migration to occur between “connected” factor markets
$\overline{FS}_f$	Factor supply in initial equilibrium
$gles_c$	Share of government exp. allocated by commodity
$htax_c$	Household tax rate
i	Discount rate
$itaxac_{af}$	Indirect tax rate
μ_a	Land transformation rate from arable to grassland
μ_g	Land transformation rate from arable to grassland degraded
T	Planning horizon
tm_c	Tariff rate
$wfrat_{f_1 f_2}$	Wage ratio for “connected” factor markets
$yhfc_{f,h}$	Share of factor income to household
$zles_c$	Share of investment allocated by commodity
Variables	
$ABSORB$	Total absorption
CD_c	Final demand for private consumption
$DWG_{f_1 f_2}$	Wage differential between f_1 and f_2
EXR	Exchange rate (R$ per $US)
$FDSC_{f_1 a}$	Factor demand sector
$FSAV$	Net foreign savings
FS_f	Factor supply
GD_c	Final demand for government consumption
$GDTOT$	Total government demand
GR	Government revenue
$HREMIT$	Remittances
ID_c	Final investment demand
$INVABS$	Investment to absorption ratio
$INVEST$	Total investment
MPS_h	Marginal propensity to save
PA_a	Domestic activity price

(continued)

Table 3.3—Continued

Equation	Definition
Sets	
PD_c	Domestic commodity price
PE_c	Domestic price of exports
PM_c	Domestic price of imports
PQ_c	Price of composite good
PWE_c	World price of exports
PWM_c	World price of imports
PX_c	Average output price
$PXAC_{a,c}$	After-tax price of commodity *c* from activity of *a*
$PXACP_{a,c}$	Pre-tax price of commodity *c* from activity *a*
QA_a	Domestic activity output
QD_c	Domestic sales
QE_c	Exports
$QFCON_{f_1 f_2}$	Factor conversion from factor f_1 to f_2
$QFMIG_f$	Net migration of factor *f*
QM_c	Imports
QQ_c	Composite goods supply
QX_c	Domestic commodity output
$QXAC_{a,c}$	Domestic output of commodity *c* from activity *a*
$SAVING$	Total savings
$UESH_f$	Share of factor *f* going unemployed
$WF_{f,a}$	Sectoral factor price
$WFAVG_f$	Average factor price
$YFCTR_f$	Factor income
YH_h	Household income
Functional dependencies	
CES	Constant elasticity substitution
CET	Constant elasticity of transformation
TRANSLOG	Translogarithmic flexible functional form
*FOC*1	First order condition (FOC) for CES production
*FOC*2	FOC for translog commodity production
*FOC*3	FOC for CET transformation between products for export and domestic markets
*FOC*4	FOC for CES substitution in consumption between import goods and domestically produced goods

Note: See Table 3.4 for the equations for the simplified model and Appendix A, Tables A.1 and A.2 for the full CGE model.

Table 3.4 Model description (simplified version with no intermediate goods)

Equation	Description of equation
Price equations	
1. $PM_c = PWM_c \cdot (1 + tm_c) \cdot EXR$; $PE_c = PWE_c \cdot EXR$	Import price and export prices
2. $PQ_c = \dfrac{PD_c \cdot QD_c + PM_c \cdot QM_c}{QQ_c}$	Composite commodity prices
3. $PX_c = \dfrac{PD_c \cdot QD_c + PE_c \cdot QE_c}{QX_c}$	Composite producer prices
4. $PXAC_{a,c} = PXACP_{a,c} \cdot (1 + itaxac_{a,c})$	Commodity prices (including indirect taxes)
5. $PA_a = TRANSLOG\left(PXACP_{a,c}, Q_a\right)$	Activity prices(multi-output activities)
Quantity equations	
6. $QA_a = CES(FDSC_{f,a})$	Activity production (CES)
7. $\dfrac{FDSC_{f,a}}{QA_a} = FOC1(WF_{f,a}, PA_a)$	Demand for primary factors
8. $QX_c = CES(QXAC_{a,c})$	Commodity demand(CES aggregation)
9. $\dfrac{QXAC_{a,c}}{QX_c} = FOC2(PX_c, PXAC_{a,c})$	Disaggregated commodity demand
10. $QA_a = TRANSLOG(QXAC_{a,c})$	Activity production (translog aggregation)
11. $\dfrac{QXAC_{a,c}}{QA_a} = FOC2(PX_a, PXACP_{a,c})$	Disaggregated multi-commodity production by activity a.
12. $QX_c = CET(QE_c, QD_c)$	Output transformation (CET) for exporting sectors
13. $\dfrac{QE_c}{QD_c} = FOC3(PE_c, PD_c)$	Export supply for exports
14. $QQ_c = CES(QM_c, QD_c)$	Armington assumption:Composite commodity aggregation (CES)
15. $\dfrac{QM_c}{QD_c} = FOC4(PM_c, PD_c)$	Import demand

(continued)

Table 3.4—Continued

Equation		Description of equation
Income equations		
16.	$YH_h = \sum_{f \in F} \sum_{a \in A} yhfc_{f,h} \cdot WF_{f,a} \cdot FDSC_{f,a} + HREMIT_h$	Household income
17.	$GR = \sum_{h \in H} htax \cdot YH_h + \sum_{a \in A} itaxac_{a,c} \cdot PXACP_a \cdot QXAC_{a,c} + \sum_{c \in C} tm \cdot PWM_c \cdot QM_c \cdot EX$	Government revenue
18.	$SAVING = GOVSAV + FSAV \cdot EXR + \sum_{h \in H} MPS_h \cdot YH_h \cdot (1 - htax_h)$	Total savings
Expenditure equations		
19.	$PQ_c \cdot CD_c = \sum_{h \in H} cles_c \cdot (1 - mps_h) \cdot (1 - htax_h) \cdot YH_h$	Household consumption demand
20.	$GD_c = gles_c \cdot GDTOT$	Government consumption demand
21.	$ID_c = zles_c \cdot INVEST$	Fixed investment demand
Factor supply and demand, and migration relationships		
22a.	$FS_{f1} = \overline{FS}_{f1} + \sum_{\substack{a \in A \\ f2 \in f}} \mu_{a,f1,f2} \cdot FDSC_{f_2,a} + \sum_{f2 \in f} QFCON_{f1,f2}$	Factor supply (no migration) Includes factor transformation for physical causes and factor conversion (such as deforestation)
22b.	$FS_f = \overline{FS_f} + QFMIG_f \ for\ f \in FMIG$	Factor supply (with migration)
23.	$QFMIG_{f_1} = \sum_{f_2} (OUTMIG_{f_2,f_1} - OUTMIG_{f_1,f_2})$	Net migration arriving into f1
24.	$WFAVG_f = \sum_{a \in A} WF_{a,f} \cdot FDSC_{a,f} \Big/ \sum_{a \in A} FDSC_a$	Average factor wage
25.	$WFAVG_{f1} = wfrat_{f1,f2} \cdot (1 + DWG_{f_1,f_2}) \cdot WFAVG$	For “connected” factor markets, the wage ratio is constrained
26.	$DWG_{f1,f2} > dwt_{f1} \ \left[OUTMIG_{f1,f2} > 0\right]$	Migration occurs when wage differential exceeds threshold
27a.	$\sum_{f \in FMIG} QFMIG_f = 0$	Conservation of total factor supply (for factors that are “connected” through migration)
27b.	$\sum_{f_1,f_2 \in FCON} QFCON_{f_1,f_2} = 0$	Conservation of total factor supply (for factors that are “connected” through conversion)

(continued)

Table 3.4—Continued

Equation		Description of equation
Factor supply and demand, and migration relationships		
28a.	$FS_f = \sum_{a \in A} FDSC_{a,f}$	Factor market equilibrium (fully employed factors)
28b.	$FS_f > \sum_{a \in A} FDSC_{a,f} \quad \left[WF_f > wf_f^{\min} \right]$	Factor market equilibrium (potentially unemployed factors)
29.	$UESH_f = \left[FS_f - \sum_{a \in A} FDSC_{a,f} \right] \Big/ FS_f$	Share of factor going unemployed (for potentially unemployed factors)
30.	$PX"def" = \frac{WFAVG"ar"}{i+\mu_a}[\,1 - e^{-(i+\mu_a)T}\,] + \frac{WFAVG"gr"}{i+\mu_g}[\,1 - e^{-(i+\mu_g)T}\,] - \frac{WFAVG"gr"}{i+\mu_a+\mu_g}[\,1 - e^{-(i+\mu_a+\mu_g)T}\,] - \alpha \frac{WFAVG"for"}{i}$	Deforestation demand: price is the expected NPV of returns to land
Macroeconomic closures		
31.	$QQ_c = CD_c + ID_c + GD_c$	Commodity market equilibrium
32.	$\sum_{c \in C} PM_c \cdot QM_c = \sum_{c \in C} PE_c \cdot QE_c + FSAV + \sum_{h \in H} HREMIT_h$	External account balance
33.	$ABSORB = \sum_{c \in C} PQ_c\,(\,CD_c + ID_c + GD_c + DST_c)$	Total absorption
34a. 34b.	$\overline{GOVABS} = \frac{\sum_{c \in C} PQ_c \cdot GD_c}{ABSORB}\;;\;\overline{INVABS} = \frac{\sum_{c \in C} PQ_c \cdot ID_c}{ABSORB}$	Government consumption and investment demand (fixed share of absorption)
35.	$SAVING = INVEST$	Saving-Investment balance

Source: Compiled by author.

a certain degree of independence from export prices and dampens export responses to changes in the producer environment.

Model Specification

The definitions of the terms used in the model are listed in Table 3.3, and a simplified version of the model used for the simulations is presented in Table 3.4. To highlight the special features of the model, this version ignores intermediate demands, which are treated in a standard way in the full model.

Price Equations

The first set of equations defines prices in the model. On the import and export sides, the model incorporates the "small country" assumption, which states that world prices are exogenous. In the two parts of equation (1), the domestic price of imports and exports is the world price times the exchange rate, with domestic import prices also including a price wedge expressed by the import tax rate (tm_c). The prices of composite commodities (made up of imports and commodities from domestic producers) are defined as a weighted average of domestic and imported commodity prices adjusted for the consumption tax (equation 2). In a parallel manner, for any commodity, the aggregate producer price is a weighted average of domestic sales and export prices (equation 3). The model makes a distinction in equation 4 between price paid for a commodity to an activity based on whether it is at producer prices ($PXACP_{a,c}$) or whether it includes indirect taxes ($PXAC_{a,c}$). The (gross) price paid for any activity (revenue per unit of the activity) is a function of output and commodity prices (equation 5).

Quantity Equations

Equations 6–15 show the quantity equations for commodities and factors that are related to production and foreign trade (the latter only for commodities). Equation 6 defines the CES production function, which, for each activity, determines the relationship between the quantity produced and the use of primary factors. Equation 7 is the demand function for factors, derived from the first-order condition for profit maximization subject to equation 6. Equation 8 defines the demand at the national level for the commodities produced at the regional level. Equation 9 is the first order condition for cost minimization and captures competition between multiple activities (distinguished by their specific technologies) producing a single commodity. Outputs from different activities are imperfect substitutes, an application of the Armington approach (commonly used for international trade) in a domestic setting. In addition to the standard one-to-one mapping between activities and commodities, equation 10 permits multiple outputs for any given activity. More specifically, equation 10 defines aggregate output as a translogarithmic function of output disaggregated by the commodity produced. Equation 11 is a first-order condition derived from cost-minimization subject to equation 10 and a fixed aggregate output demand level. This approach is particularly useful in the context of this project to take into consideration the ease or difficulty farmers have in shifting production from one crop to another.

Equation 12 provides the CET function that transforms domestic output to commodities for exports and domestic sales. Equation 13 is derived from profit maximization subject to equation 12 and a fixed level of domestic output; it defines export supply as a function of relative prices. Equation 14 shows how imports and domestic output sold domestically generate the composite commodities that are demanded by all domestic users. Equation 14 is the Armington function, which is the CES aggregation function for imports and domestic output sold domestically. Equation 15 gives the import demand functions of the relative prices of imports and domestic commodities; it is derived from cost minimization, subject to equation 14 and a fixed level of composite commodity demand. Figure 3.5 summarizes the flow of commodities from production activities to the domestic market and exports. It should be noted that the commodities *QXAC*, *QX*, *QD*, and *QE* are distinct and associated with separate prices (*PXAC*, *PX*, *PD*, and *PE*, respectively). Imports (*QM*) and domestic goods (*QD*) are also distinct from their composite (*QQ*) with separate sectoral prices.

Income Equations

Institutional income flows are extremely simplified in this reduced version. The model institutions are households, government, the savings/investment account, and the rest of the world. Factor income, as a function of factor demand and factor prices, is channeled to the households, and remittances from abroad are also assigned to households (equation 16). Government revenue is defined in equation 17 as the sum of revenue from household taxes, indirect taxes, and import taxes. Total saving, defined in equation 18, is made up of government savings, foreign savings, and household savings.

Expenditure Equations

Domestic final demands are composed of private consumption and investment demand. For each household, consumption is determined by a Cobb-Douglas function, distributing marginal budget share across commodities (equation 19). Similarly, equations 20 and 21 assure that government demand and investment demand are, respectively, allocated across commodities in fixed value shares.

Factor Markets, Migration, and Unemployment

The supply of nonmigrating factors depends on the initial stock, physical transformation, and conversion (equation 22a). Transformation is allowed from arable land to pasture/grassland and from grassland to degraded land. Conversion is allowed from forested land to arable land, and from unemployed arable land to pasture/grassland. In the long run scenarios, interregional mobility of labor and rural capital is assumed. This entails updating factor stocks (equation 22b) based on the balance of in-migration and out-migration for the factor (equation 23). Migration is assumed to rise when there are interregional differences in factor wages, therefore, the average wage of a factor over all activities in which it is employed is defined in equation 24. Keeping in mind that factors are differentiated based on whether they are employed in urban sectors or employed regionally for agriculture, migration is required to maintain the wage ratio between regions in a reasonable range. This is expressed in equation 25 where the wage ratio imposed between two factors is in the neighborhood of a fixed value $wfratf_1,f_2$. The neighborhood of variation for the wage ratio is defined in equation 26, which is specified as a mixed complementarity problem: the wage differential is written as an inequality and linked to the migration variables in the complementary slackness conditions. To allow for interregional differences in the propensity to migrate, the wage differential threshold in the inequality (below which migration does not occur) depends on both the receiving factor (f_1) and the factor providing the migrant flow (f_2). To conclude the migration block, equations 27a and 27b express the conservation of factors, meaning that the net migration and conversion of factors summed over all factors have to balance out to zero.

The equilibrium conditions for factor markets are defined in equations 28a and 28b. It is assumed in the short run that all factors except capital may go unemployed. In the long run only arable land may go unemployed, in which case it is converted to grassland/pasture. Flexible average factor prices perform the task of equilibrating each market. In equation 29, if the lower bound for a factor price becomes binding, a share of the factor will not be employed (*UESHf*). To the extent that it is demanded by different sectors, each factor of production is assumed to be sectorally mobile inside its region.

To conclude the section on factors, the demand for deforestation (producing arable land), expressed by equation 30, will be derived in detail in a later section dedicated explicitly to quantifying the demand for deforested land. In general terms, it expresses the price for arable land as being determined by the returns to agricultural land, which is in turn affected by land degradation. For tenured land, the net returns to deforestation will also depend on the profitability of standing forest (last term in equation 30).

Macroeconomic Closure

Equation 31 is the equilibrium condition for composite commodity markets: supply is set equal to the sum of final demands; flexible composite commodity prices assure that this condition is satisfied. Equation 32 specifies the equilibrium condition for the current account of Brazil's balance of payments. The domestic price index is chosen as numeraire. Foreign savings is fixed (the current account deficit), and the real exchange equilibrates the current account. Absorption is defined in equation 33 as the sum of final demands (investment and government and consumption spending). This definition is drawn upon in equation 34a, which determines the nominal values of investment spending as a fixed share of absorption, and in equation 34b, which similarly determines government spending. Equation 35 defines the final macro closure condition, imposing equality between the values of total savings and total investment.

Demand for Deforested Land

The price for arable land, P_{ar}, is determined by the returns to agricultural land. In an infinite horizon framework, the flow return from an asset divided by the asset price must be equal to the rate of interest in the steady state. What is obtained by going down this path is a perfectly elastic demand for cleared land (which is a reasonable assumption since the investment in newly cleared land is a negligible share of aggregate investment). This implies that the price of arable land for a squatter, *assuming a fixed rental rate*, would be

$$P_{ar} = \int_0^T r_{ar}\, e^{-it}\, dt = \frac{r_{ar}}{i}[\,1 - e^{-iT}\,]$$

This expression takes into consideration that an agricultural producer's decision to buy arable land depends on the tenure regime: if the land is subject to insecure property rights, the planning horizon will be finite. A limitation of the expression is that it does not take into account that the rental rate may vary with time due to decreasing or increasing productivity.

For the purpose of this analysis, it is reasonable to assume that arable land is transformed through degradation to grassland, which can be used only for pasture. Let the degradation rate equal μ_a (the indices are dropped to simplify notation) and let r_{gr} equal the rental rate of grassland, then the price for 1 hectare of newly deforested land, if the planning horizon is assumed to be T, is given by the following equations.

Assume $\frac{dA_{ar}}{dt} = -\mu_a\, A_{ar}$

so that $A_{ar} = A_0\, e^{-\mu t}$ with $A_0 = 1$ (hectare),

then
$$P_{ar} = \int_0^T r_{ar}\, e^{-it} \cdot e^{-\mu_a t} + r_{gr}\, e^{-it} \cdot (\,1 - e^{-\mu_a t}\,)\, dt$$

the solution being:

$$P_{ar} = \frac{r_{gr}}{i}[\,1 - e^{-iT}\,] + \frac{(\,r_{ar} - r_{gr}\,)}{i}[\,1 - e^{-(\,i + p\,)T}\,]$$

The interpretation of the last equation is straightforward: the first term represents the value derived from the use of one hectare of land before it degrades to grassland; the second term represents the value derived after conversion to grassland. If there is no land transformation, r_{gr} drops out of the third equation above and the expression simplifies into the first equation. As the degradation rate, μ_a increases the value of a hectare of arable land approaches that of a hectare of grassland.

The above expression, however, does not take into account that the use of grassland for livestock purposes is not agronomically sustainable in many regions of the Brazilian Amazon. To take this additional degradation process into consideration, one must proceed in a manner similar to that adopted to compute the effect of degradation of arable land: if grassland area in livestock use degrades exponentially (after being generated through transformation of arable land) according to

$$\frac{dA_{gr}}{dt} = -\mu_g\, A_{gr}$$

then the expression for the price of newly arable land becomes

$$
\begin{aligned}
P_{ar} &= \int_0^T r_{ar} e^{-it} \cdot e^{-\mu t} + r_{gr} e^{-it} \\
&\quad \cdot (1 - e^{-\mu_a t}) e^{-\mu_g t} dt \\
&= \frac{r_{ar}}{i+\mu_a} [1 - e^{-(i+\mu_a)T}] \\
&\quad + \left[\frac{r_{gr}}{i+\mu_g} [1 - e^{-(i+\mu_g)T}] \right. \\
&\quad \left. - \frac{r_{gr}}{i+\mu_a+\mu_g} [1 - e^{-(i+\mu_a+\mu_g)T}] \right]
\end{aligned}
$$

This is the expression for the price of arable land used in the simplified model (first three terms in equation 30 in Table 3.3). The first term, expressing the value derived before transformation to grassland, has not changed. What has changed, as one would expect, is the value derived from use after conversion to grassland: the limited returns resulting from the degradation of grasslands have now been factored in. As a special case, if μ_g is equal to zero (no grassland degradation) then the equation reverts to the previous case.

The deforesters, being the suppliers of arable land, are faced with this price and the amount of land that will be deforested will depend on P_{ar}, on the returns from forested land (last term in equation 30), and on the squatters' profit-maximizing behavior and technology. The behavior of agents carrying out the land clearing can be differentiated according to whether the forest is an open-access resource or whether property rights governing the use of the forest resource are well defined. For the purpose of this report, it is assumed that the returns to the deforestation activity are based both on acquiring property rights to unclaimed land and on the future returns to agriculture. The net returns to deforestation are different depending on whether land is titled or not; if the land is tenured one must subtract the returns from forested land in the computation. An average return is computed here by taking into consideration that about one-third of agricultural land in the Amazon has been reported to involve fraudulent titles (Brazil, Ministry of Agrarian Development 1999). Therefore, the parameter indicating the share of deforestation occurring on tenured land (α in equation 30) is assumed to equal $^2/_3$. By assuming the planning horizon to be sufficiently long when using arable land, we allow agents to acquire property rights through deforestation.

One last complication, which has not been considered in equation 30, is that arable land can go unemployed and be used as grassland/pasture. If this happens, then the expected returns from agricultural land will be affected, as well as the price paid to deforesters for cleared land. The modified equation 30, as it appears in the full model, taking into consideration the fact that unemployed arable land earns returns equal to those of grassland pasture, is

$$
\begin{aligned}
P_{ar} = (1 - UESH_{ar}) & \left[\frac{r_{ar}}{i+\mu_a} [1 - e^{-(i+\mu_a)T}] \right. \\
& + \frac{r_{gr}}{i+\mu_g} [1 - e^{-(i+\mu_g)T}] \\
& \left. - \frac{r_{gr}}{i+\mu_a+\mu_g} [1 - e^{-(i+\mu_a+\mu_g)T}] \right] \\
& + UESH_{ar} \left[\frac{r_{gr}}{i+\mu_g} [1 - e^{-(i+\mu_g)T}] \right]
\end{aligned}
$$

The equation takes into consideration that the land will be transformed gradually into grassland/pasture (every period a share of land, μ_a, is transformed to grassland). If arable land is fully employed ($UESH_{“ar”} = 0$), the last term in the equation is zero, and the equation reverts to Equation 30. If arable land is not fully employed, $UESH_{“ar”} > 0$, the price of newly cleared land is a weighted average of the price of grassland and arable land, and as the share of unemployed land increases, the returns to arable land approximates that of grassland. In the extreme case, where all arable land goes unemployed ($UESH_{“ar”} = 1$), all newly cleared land will be used as pasture; in this case the price of a hectare of deforested land equals the net present value of a hectare of grassland.

Biophysical Component

This research considers the biophysical processes related to crop sustainability. Among these processes are some that can substantially reduce agricultural productivity, such as soil degradation and weed infestation—problems that usually appear after the first few cropping cycles when a plot is cleared. The biophysical component of the modeling framework affects the equilibrium stocks of the different land types by computing the extent of land transformation given the land uses arising from the simulation. This framework is a first step in linking biophysical changes that occur with a certain land use to the economic incentive for agents to modify existing land use patterns. Including a representation of physical processes in the economic framework is important, because these processes are a major constraining factor for regional development in the Amazon region.

Different productive activities will have different effects on land quality over time. This process belongs to a class of problems that has been studied extensively in the research area known as landscape ecology (Shugart, Crow, and Hett 1973; Horn 1975; Baker 1989; Acevedo, Urban, and Ablan 1995). This research attempts to exploit the analogy between the models developed in landscape ecology, which focus on the succession of ecological states, and the current analysis of the succession of land types given existing land use.

A variety of criteria could be used to distinguish models of land-type change. Perhaps the two most important are the level of aggregation and the use of continuous or discrete mathematics. Models could also be distinguished by the kind of data sources, the method of defining states, and a number of other criteria. A critical research choice that will have to be made early in the research process is the definition of land types. This will likely vary by agro-climatic region. For example, in forest areas and at the forest margins, a possible disaggregation of land types would include pristine forest, arable land, grasslands, and degraded lands, with each type further divided into rich soil and poor soil. In lowland agriculture, the basic division could be between irrigated and rainfed land, with these types further subdivided by soil nutrient status.

The level-of-aggregation criterion refers to the level of detail with which the process leading to changes in land type is modeled. Baker (1989) describes three kinds of models. First are *whole landscape models,* in which the value of a variable in some region is modeled. Second are *distributional landscape models,* in which the distribution of values of a variable in some region is modeled. For example, taking all the land in the region under analysis, one might model the number of hectares falling in each land category (thus losing the differentiation by location). Finally, in the most detailed form are *spatial landscape models,* where the outcome of individual subareas of the landscape and their configuration are modeled. In such spatial models, for example, one could consider the number of hectares in each land category for each farm in the region (ideal if GIS data are available). For this study, a distributional model applied to land types under a given land use is attempted. This choice is necessary because the economic counterpart will consider land use decisions at a regional scale.

Both continuous and discrete mathematics have been used for the time dimension in these models, but there may be little difference in the application of these two approaches. For example, the average response of a stationary Markov process can be obtained by using the corresponding linear constant-coefficient differential equation (Shugart, Crow, and Hett 1973).[27] The matrix approach may still provide an easier framework for modeling changes in variance along with changes in mean. In most cases, empirically based models use estimates of change determined by resampling the landscape at discrete time intervals. The model is in discrete time and the intervals considered are years. The state space is also discrete because a finite number of states in which land can be classified are considered.

Assuming that the process that affects land quality through land use can be described by a land transformation matrix for any farm plot, which can be defined as

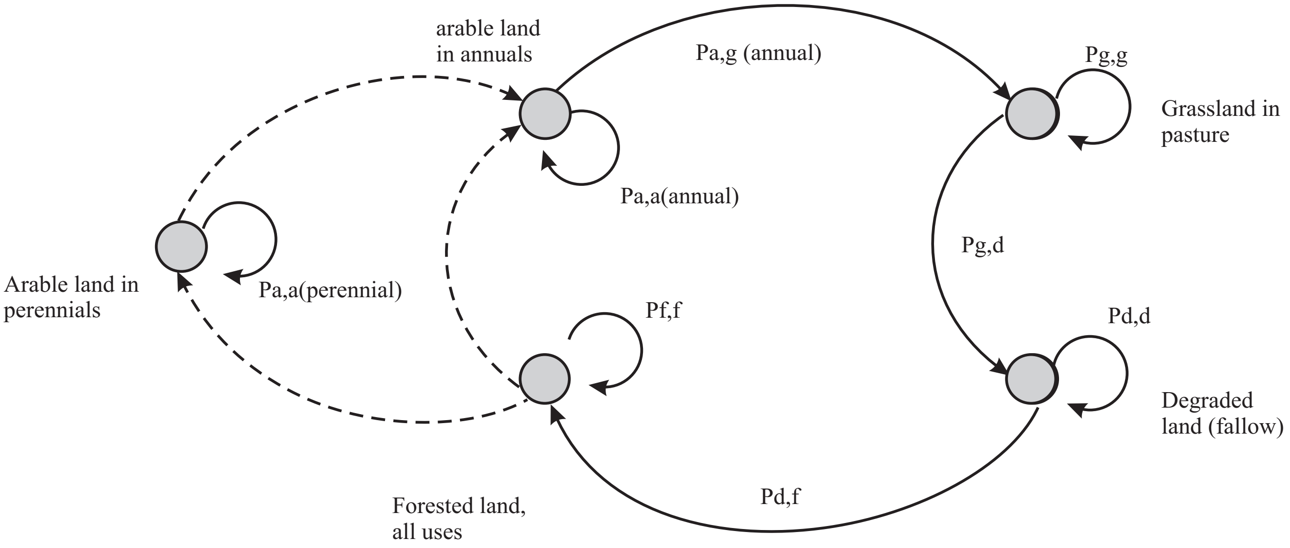
$$P = \left\{ p_{fgi} \right\} \quad f,g = 1,2,\ldots,m$$
$$i = 1,2,\ldots,n$$

where p_{fgi} is the conditional probability that an area of land of type *f* will be transformed into land of type g under activity i between two points in time. Initially, the dependence of the probabilities on the plot's history of land use will be ignored.

The above specification at the plot level is not useful in the context of a model where the unit of analysis is a region like the Brazilian Amazon. To perform the necessary leap in geographic scale, the assumption is made that the regional land stocks by type follow the same transformation pattern.[28] Let L_t be a row vector that specifies total hectares in each land type at time t, then

$$L_{t+1} = L_t P,$$

Figure 3.6 Markov chain representation of biophysical transformation processes



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[27] This equivalence in treatment is utilized in this report to the extent that the problem is presented in discrete time for estimation purposes of the transition probabilities, but the representation in the model adopts a continuous time specification (equation 30 in Table 3.3).

[28] Extrapolating from plot level data to Amazon-wide processes can be justified for these biophysical phenomena because, at the simplified level of analysis, the plot differences average out, leaving the important Amazon-specific characteristics that define the problem relative to the rest of Brazil.

where L_{t+1} is a projection at time t+1 of land stocks by type as predicted by the model of the physical transformation process. This is not to be confused with the conversion of land arising from rental differentials in the CGE model. The former expresses a natural physical process (given a fixed land use), while the latter embodies a decision by economic agents to put land to a different use, requiring a physical conversion in land type. With respect to the land types specified in the CGE model, the feasible transformations and conversions are expressed in Figure 3.3.

Technically, the natural transformation process is modeled as a first-order stationary Markov process, with land use entering as an exogenous variable (Baker 1989; Burnham 1973).[29] As shown in Figure 3.6, the system has four biophysical states (forested land, grassland, degraded land, and arable land); the latter, arable land, is divided into two different “exogenous” uses—annual or perennial cultivation—for a total of five states. The probability of remaining in or leaving a particular state is shown in association with the respective arcs ($p_{a,a,}$. $p_{a,g}$, $p_{,g,g}$, $p_{g,d}$, $p_{d,d,}$ $p_{d,f,}$ $p_{f,f}$). The probabilities are assumed to be constant for all times into the future. The dashed arcs have no associated probabilities because they are linked to economic decisions that are exogenous to the biophysical component.

The Markov chain approach is reconciled with the static CGE approach by assuming that over an area like the Brazilian Amazon, the probability of transformation can be assumed to correspond to the average transformation. So for example, if $p_{a,a}$ (annuals) in Figure 3.6 is equal to 0.33, this means that at any time 33 percent of the arable land in annuals is being transformed to grassland. Unless there are unexpected shifts in sectoral production from one year to the next, which is improbable, approximating the transformation processes by using the expected value is a valid approach. The probabilities presented here can therefore be used to obtain flows between the stocks of different land types, thereby affecting the equilibrium level of land as a factor of production (equation 22a in Table 3.3). The model solves simultaneously for these flows and for the production pattern, which reflects agents’ correct expectations about land degradation processes.

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[29] A Markov process is one that describes a stationary stochastic process with discrete, identifiable states, where the future state of the system depends only on the state immediately preceding it.